A MESHLESS METHOD WITH ENRICHED BASIS FUNCTIONS FOR SINGULARITY PROBLEMS

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For the last several decades, the Finite Element Method (FEM) has been a powerful tool in solving challenging science and engineering problems, especially when solution domains have complex geometry. However the mesh refinements and construction of higher order interpolation fields were prominent difficulties in classical finite element analysis.

In order to alliveate the difficulties of classical finite element method, the meshless methods were introduced. Meshless methods appear in several different names such as Element Free Galerkin Method (EFGM), h-p cloud Method, Partition of Unity Finite Element Method (PUFEM), Generalized Finite Element Method (GFEM), and Extended Finite Element Method (XFEM). In this dissertation, we are concerned with enriched GFEM. Unlike classical finite element methods, these meshless methods use meshes minimally or not at all. This feature becomes powerful when it comes to model crack propagation, large deformation, etc because re-meshing is unnecessary.

A partition of unity is an essential component of GFEM. The partition of unity function employed in this dissertation, is unique in the following sense: First, the partition of unity functions are highly regular, whereas most GFEM in the literature use piecewise C^0-partition of unity functions. The highly regular partition of unity functions with appropriate smooth local approximation functions enables us to have highly regular global basis functions. Second, if polynomial local approximation functions that satisfy the Kronecker delta property are chosen, the global basis functions become smooth piecewise polynomials, and hence numerical integrations become exact and imposing essential boundary conditions become simple. Third, the partition of unity shape functions designed to have flat-top do not yield an ill-conditioned stiffness matrix. Furthermore, a partition of unity for a non-convex domain is introduced to deal with an elasticity problem on a cracked elastic medium.

The most powerful aspect of GFEM is the freedom to choose any desired local approximation functions. By choosing highly smooth local basis functions, it would be possible to solve high order PDEs such as biharmonic and polyharmonic partial differential equations without using Hermite finite elements that are extremely difficult to implement. Moreover, when a given problem has strong singularities, using various types of singular functions, the approximation space can be enriched to capture the singularities without regenerating the whole mesh or refining the meshes in the adaptive way.

In this presentation, GFEM with enriched basis functions is demonstrated to solve elliptic boundary value problems containing singularities. Two numerical examples will be given: The Motz problem that has jump boundary data singularity and highly accurate stress analysis of cracked elastic domains. We demonstrate that the proposed approach yields highly accurate numerical solution of the Motz problem as well as accurate stress analysis of cracked elastic domains. We also will show that the meshless method, GFEM with enriched basis functions, yields the improved results, compared with performance of other existing methods. Finally, we introduce a new approach to estimate the stress intensity factor.